

Yale University

EliScholar – A Digital Platform for Scholarly Publishing at Yale

Cowles Foundation Discussion Papers

Cowles Foundation

3-1-1988

Warranties as Signals under Consumer Moral Hazard

Nancy A. Lutz

Follow this and additional works at: <https://elischolar.library.yale.edu/cowles-discussion-paper-series>



Part of the [Economics Commons](#)

Recommended Citation

Lutz, Nancy A., "Warranties as Signals under Consumer Moral Hazard" (1988). *Cowles Foundation Discussion Papers*. 1110.

<https://elischolar.library.yale.edu/cowles-discussion-paper-series/1110>

This Discussion Paper is brought to you for free and open access by the Cowles Foundation at EliScholar – A Digital Platform for Scholarly Publishing at Yale. It has been accepted for inclusion in Cowles Foundation Discussion Papers by an authorized administrator of EliScholar – A Digital Platform for Scholarly Publishing at Yale. For more information, please contact elischolar@yale.edu.

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 867

Note: Cowles Foundation Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. Requests for single copies of a Paper will be filled by the Cowles Foundation within the limits of the supply. References in publications to Discussion Papers (other than acknowledgment that a writer had access to such unpublished material) should be cleared with the author to protect the tentative character of these papers.

WARRANTIES AS SIGNALS UNDER CONSUMER MORAL HAZARD

Nancy A. Lutz

March 1988

WARRANTIES AS SIGNALS UNDER CONSUMER MORAL HAZARD

Nancy A. Lutz
Yale University

August 1987
revised November 1987

This paper is based in part on Chapter 3 of my dissertation. My thanks to Jeremy Bulow, David Baron, and most especially John Roberts for their advice and comments. I also wish to thank Al Klevorick, Russ Cooper and seminar participants at Stanford, Iowa, and Yale.

Section 1: Introduction

The economic purpose of product warranties seems at first glance to be obvious: warranties provide insurance against unsatisfactory product performance. As long as the purchasers of any product are risk averse, providing them with such insurance may be profitable. Indeed, if the insurer is risk neutral, warranties should provide full insurance. Many products, however, are sold with warranties providing only incomplete insurance: the typical warranty obligates the manufacturer to repair the product for a limited period, perhaps a year.

One reason for incomplete warranties is that manufacturers as well as consumers may be risk averse. Heal (1977) addresses how risk is optimally allocated through a warranty when both the producer and the consumer of a product are risk averse. Another explanation is that warranty insurance like any other kind of insurance is subject to moral hazard. As long as the consumer of a product makes choices that affect the likelihood that the product will break down, insurance against breakdown will change the consumer's incentives and therefore his choices. In particular, if the consumer can take some costly action to reduce the probability of breakdown, a warranty reduces the incentive to take the action. Such a costly action might be verifiable: when it is, the warranty contract can be conditioned on the action taken. An example would be automobile warranties, which specify the maintenance required: oil changes and the like. But we cannot expect that all such actions are verifiable. It may be impossible for the manufacturer or any third party to observe that your TV set is broken because your child kicked it, or your telephone no longer works because you dropped it repeatedly, or your space heater burned out after you neglected to oil the motor. When such an unverifiable action exists,

we are faced with the possibility of consumer moral hazard. This kind of moral hazard limits the incentive for warranty provision: even when the producer is risk neutral and the consumer risk averse, such moral hazard will result in incomplete warranties.

Beside risk-sharing, there are two other incentives for warranty provision that have been examined in the literature. One is producer moral hazard. This exists when the actual probability that a given unit of a product will break down is unobservable but inferred from earlier experience with the product, or is specified by contract. The unobservability means that the producer may have an incentive to cheat, producing a unit of less-than-contracted or less-than-prior quality. A warranty reduces this incentive as long as lower quality products are more likely to break down. The combined effects of producer and consumer moral hazard have been analyzed by Cooper and Ross (1985). The second incentive is product differentiation. The identical products may be offered with different warranties to allow a monopolist to price discriminate between consumers who vary in their willingness to pay for insurance (Matthews and Moore, 1987). Duopolists may compete on both price and warranty terms (Lutz, 1986).

Warranties, besides providing insurance, may also be a way for the firm to communicate information about product quality to consumers. Whether the empirical evidence supports this possibility is a matter of some dispute. It has been claimed that warranties do not serve as signals of product quality because high quality goods are not always or even usually sold with higher warranties than low quality goods. For example, Priest (1981) surveys warranties offered in 1974 on seven different kinds

of appliances and finds no clear relationship between the duration of the warranty and the expected life span of the product. He concludes that this evidence provides weak support at best for any signaling role for warranties. This conclusion is based on evidence across products, but studies of warranty terms on different brands of the same kind of appliance have reached the same conclusion.¹

Whether or not warranties do serve as signals of product quality, they have all the characteristics of a signal. By federal law--the Magnuson-Moss Warranty Act of 1974--warranties must be simply and clearly written, conspicuously disclose all important provisions, and must be available for inspection at the point of sale before the product is purchased.² In economic terms, warranties must be observable before purchase.

As long as a high quality product is less expensive to warrant than a low quality product, we might expect that the observable warranty could serve as a signal of product quality. In a model without consumer moral hazard, Grossman (1981) analyzes how a monopolist chooses the warranty to be bundled with a product of exogenous fixed quality when the single consumer cannot observe the quality before purchase. There is a wide variety of equilibria to this game, but Grossman applies a refinement to the consumer's beliefs off the equilibrium path and finds a unique equilibrium satisfying this refinement. This is a pooling equilibrium, but it is unusual because in this equilibrium the monopolist offers the product with a full warranty and charges the reservation price. With such complete

¹Gerner and Bryant (1981) and Bryant and Gerner (1978).

²Magnuson-Moss Warranty--Federal Trade Commission Improvement Act §§101-112, 88 Stat. 2183 (1975) (codified at 15 U.S.C. §§2301-2312 (1976)).

insurance against breakdown, the consumer is indifferent to product quality and hence his reservation price for the product sold with this warranty does not depend on its quality. This pooling equilibrium contract is the contract that the monopolist would offer if its quality were observable: when this is true, profit maximization calls for full insurance at the reservation price.

This result is difficult to reconcile with the empirical facts that have led some to conclude that warranties do not serve as signals. We do not generally observe the full warranties predicted in this model. However, this conclusion about the empirical importance of signaling with warranties rests on the false premise that in a signaling model higher quality necessarily implies a higher warranty in equilibrium.

In this paper, I examine whether and how warranties serve as signals of product quality in an environment where there are opportunities for consumer moral hazard. My model is very similar to Grossman's. A risk neutral monopolist produces a good of fixed and exogenous quality. This product is offered to a market of identical risk-averse consumers, and it can be bundled with a warranty of the monopolist's choosing. The probability that the product breaks down is a function of its quality and the effort the consumer takes in using it. This consumer effort cannot be observed by the monopolist or any third party, so that the warranty cannot be made conditional on the effort taken, and in choosing the warranty the monopolist must take the moral hazard problem into account.

Section 2 presents the model and analyzes the decisions faced by the consumer and the firm under the assumption that the consumer can observe product quality before purchase. The profit-maximizing price and warranty

are determined; I find that a monopolist that produces a high quality product may not choose a higher warranty than it would select if it produced a low quality product. I then present the game played by the monopolist and the consumer when only the monopolist knows the quality of the product it sells. I define the equilibrium concept, sequential Nash equilibrium subject to a refinement on the consumer's beliefs off the equilibrium path. I also discuss under what conditions the monopolist's payoff in the game--its profit--is a monotonic function of the consumer's beliefs about product quality. Whether or not it is monotone depends on how the consumer chooses effort as a function of his beliefs. This, in turn, depends critically on how quality and effort interact to determine the probability of breakdown. Whether or not payoffs are monotonic affects equilibrium behavior.

In Section 3 I impose an assumption that guarantees payoff monotonicity, and then solve the game. Under this assumption, no pooling equilibrium satisfying the refinement exists. A separating equilibrium satisfying the conditions may exist, and if it does, it is unique. In this equilibrium the monopolist producing high quality signals, both through the warranty it offers with the product and through the price it charges for the product-warranty bundle. Whether the monopolist produces high or low quality it offers only partial insurance against breakdown. And if it produces high quality, it may signal that fact with a price-warranty contract that includes a less complete warranty than the one it would offer if it sold a low quality product.

In Section 4 the assumption made in the previous section is reversed, so that the monopolist's profits from offering any price-warranty contract can be a non-monotonic function of the consumer's beliefs about product

quality. Now there may be multiple pooling equilibria satisfying the refinement. There may also be a unique separating equilibrium. In this separating equilibrium, the monopolist may "signal" if it produces low quality. Whether or not this is so in equilibrium, high quality need not be sold with a more extensive warranty than low quality. These results suggest that the conclusions drawn about the empirical importance of warranties in transmitting information about product quality have been based on a false premise about how a warranty can signal quality. The last section expands on this and related conclusions.

Methodologically this paper concerns a multivariate signaling model. Under the assumption made in Section 3 the game considered is similar to the one analyzed in Milgrom and Roberts' (1986) article on price and dissipative advertising as signals of product quality. Both models feature the use of two signals and both analyses rely on the employment of a refinement. Milgrom and Roberts' work uses sequential elimination of dominated strategies and the Intuitive Criterion (Cho and Kreps, 1987), and this technique can be used in Section 3 as well: using it permits weakening of the conditions for the existence of an equilibrium, but changes none of my other results. But this response to the existence of multiple equilibria would be far less successful in Section 4, because of the non-monotonicity featured there: the Intuitive Criterion lacks "bite" under this condition.

Section 2: Optimal Warranties When Quality Is Observable

This section presents a model of warranty provision in the spirit of Grossman but with the addition of consumer moral hazard. Consider a consumer who must decide whether or not to purchase a product offered by a single firm. When the consumer's utility from the purchase is a random variable, the consumer's decision depends on three things: the product's price, the expected utility from the purchase, and the consumer's reservation utility.

There are many reasons why the consumer's after-purchase utility could be a random variable. One is probabilistic product breakdown, and the simplest way to model this is to assume that there are only two possible outcomes: the product performs as promised or it does not perform at all. A warranty then provides insurance against the second outcome, product breakdown or failure. I assume that the probability that the product works, which I write as π , is a function of two things: a parameter q that represents the product's intrinsic quality or durability and a variable m that represents the level of maintenance and care in use provided by the consumer. The probability that the product works is then $\pi(q,m) \in [0,1]$. Product quality is assumed to be determined by nature and is thus exogenous to the firm. It takes one of two values, either "low" or "high." Thus $q \in \{L,H\}$ with $L < H$. The probability that nature chooses high quality is $r^N \in [0,1]$. The marginal cost of production is constant and does not depend on quality; without loss of generality, I can normalize this cost to zero.

The actual quality of the product is observable to the firm, but not to the consumer, who must rely on his beliefs about quality when deciding

both whether or not to purchase the product and the level of effort m taken in its use. He knows that quality must be either low or high, so that his beliefs are given by r , his subjective probability that the product is of high quality. Before observing price or warranty, $r = r^N$. He also knows how he can affect the probability that the product works by choosing a level of m . I assume that $\pi_m > 0$ and $\pi_{mm} \leq 0$: that is, increasing maintenance increases the probability of success, but at a non-increasing rate. If maintenance were costless to the consumer, he would choose the maximum possible level of maintenance. If maintenance were observable by the firm (and verifiable), it could write a contract obligating the consumer to take a specified level of effort as a condition of purchase or equivalently of the warranty.³ I assume that effort is both costly and unobservable; the firm will have to consider the effect of warranty insurance on the consumer's incentive to take care.

The consumer who purchases the product chooses effort to maximize his utility. I can simplify this effort-choice problem by assuming that the consumer must pay for the product in full when it is purchased. Restricting access to credit markets in this way implies a two period structure for the model. In the first period, the consumer decides whether or not to purchase the product. In the second period, the consumer first chooses effort, the product then either works or breaks down, and the consumer enjoys a level of utility that depends on whether or not the product worked, the maintenance effort taken, and the warranty.

³ Automobile warranties are the obvious example in which the provision of insurance by the producer against breakdown is conditional on maintenance. But note that the maintenance required is observable, either directly--oil changes and the like--or indirectly through inspection of the broken part.

If the product works, the consumer enjoys utility $U(\theta) - g(m)$ where θ is the consumer's monetary valuation of a working product, $U(\cdot)$ is his utility function for money, and $g(\cdot)$ is his disutility for effort. I assume that $U' > 0$, $U'' < 0$, $g' > 0$, and $g'' > 0$. Hence, the consumer is strictly risk averse and values the insurance provided by a warranty. I also assume that $U(0) = 0$.

The consumer's utility in the event of a product breakdown depends on the level of warranty coverage. I assume that the consumer receives no utility from a broken product, and that the warranty takes the form of a monetary payment $w \in [0, \theta]$ to be made to the consumer if the product breaks down.⁴ The consumer utility if the product fails can be written as $U(w) - g(m)$. Then the consumer expects his utility after the product is purchased to be equal to

$$U(w) + s(r, m)\Delta(w) - g(m) \quad (1)$$

where $s(r, m) = r\pi(H, m) + (1-r)\pi(L, m)$ is the consumer's subjective probability that the product will work when he believes that the product is of high quality with probability $r \in [0, 1]$, and $\Delta(w) = U(\theta) - U(w)$ is the difference in the utility between consuming a working and a broken product, given warranty w . Note that when $w = \theta$ the warranty fully insures the consumer against breakdown, while with $w = 0$ the warranty provides no insurance.

The consumer will choose a maintenance effort m to maximize his expected utility after purchase. I write the effort taken to maintain a

⁴Alternatively, the warranty could be modeled as providing some kind of repair. The two approaches are formally equivalent, since w could then be thought of as the consumer's valuation of a repaired product.

product believed to be of high quality with probability r under warranty w as $m(r,w)$. Since expected utility is strictly concave in effort, $m(r,w)$ is unique. Clearly, $\partial m(r,w)/\partial w < 0$, so that an increase in the warranty implies a decrease in effort. This, of course, is consumer moral hazard--more warranty insurance decreases the incentive for maintenance, a form of self-insurance.

The incentive for maintenance is also affected by what the consumer believes about the product's quality, since the level of quality determines the marginal expected utility of changes in effort. We know that

$$\frac{\partial m(r,w)}{\partial r} = \frac{(-\partial^2 s(r,m)/\partial m \partial r)\Delta(w)}{(\partial^2 s(r,m)/\partial m^2)\Delta(w) - g''(m)} \quad (2)$$

and

$$\frac{\partial^2 s(r,m)}{\partial r \partial m} = \pi_m(H,m) - \pi_m(L,m) .$$

Hence, if $\pi_{mq} < 0$, effort decreases as quality increases for any given level of warranty and if $\pi_{mq} > 0$ effort increases as quality increases. Intuitively, this happens because in the first case quality and effort are substitutes in determining the probability of success--the higher are beliefs, the lower is the expected increase in the probability that the product works due to an increase in effort. In the second case, quality and effort are complements in determining the probability of success, so that higher beliefs increase the expected marginal effect of an increase in effort.

I can now address the consumer's purchase decision. Assume the consumer's income is y in the purchase period and there is no discounting.

If the consumer pays price p for a product he believes to be of high quality with probability r under warranty w , his expected total utility over the purchase period and the post purchase period is equal to

$$U(y-p) + U(w) + s(r, m(r, w))\Delta(w) - g(m(r, w)) . \quad (3)$$

The consumer will purchase the product with this warranty at this price if the total expected utility of the purchase is greater than or equal to the total utility of no purchase. Assuming that if no purchase is made the utility in the second period equals the utility from a broken product that is not under warranty, the total utility of no purchase is just $U(y)$. I can then solve for $p^M(r, w)$, the consumer's reservation price given beliefs r when the product is bundled with warranty w :

$$p^M(r, w) = y - h\{U(y) - U(w) - s(r, m(r, w))\Delta(w) + g(m(r, w))\} \quad (4)$$

where $h = U^{-1}$. This expression for $p^M(r, w)$ will be positive and less than y (so that the consumer's budget constraint is non-binding) as long as

$$U(y) > U(w) + s(r, m(r, w))\Delta(w) - g(m(r, w))$$

for all w . This is true if and only if $y \geq \theta$, as I will assume. Differentiating both sides of (4) shows that the reservation price is strictly increasing in quality for all partial warranties and is independent of quality for a full warranty. Full insurance makes the consumer indifferent about the quality of product. The reservation price is also an increasing and strictly concave function of the warranty.

Turning to the production side of the market, the monopolist's profits

will be a function of the exogenous quality of the product, the price and warranty it chooses, and the consumer's beliefs. The firm choosing price and warranty must consider both how the consumer draws conclusions about quality from price and warranty and what actions--purchase and effort--the consumer will take after reaching his conclusion. If the consumer believes r after observing p and w , then if $p \leq p^M(r, w)$ the firm earns a profit of p minus the expected warranty claims, which are defined as

$$W(q, r, w) = (1 - \pi(q, m(r, w)))w .$$

It earns nothing if $p > p^M(r, w)$, because in this case the consumer does not purchase the product.

If quality is observable by consumers, the monopolist need not worry about how its choice of price and warranty will affect beliefs. The monopolist then maximizes profits at any warranty w by charging the consumer a price of $p^M(0, w)$ if the product is of low quality and $p^M(1, w)$ if the product is high quality. Hence the monopolist's profit maximization problem can be written in terms of only the choice of warranty:

$$\underset{w}{\text{maximize}} \ p^M(r, w) - W(q, r, w) . \quad (5)$$

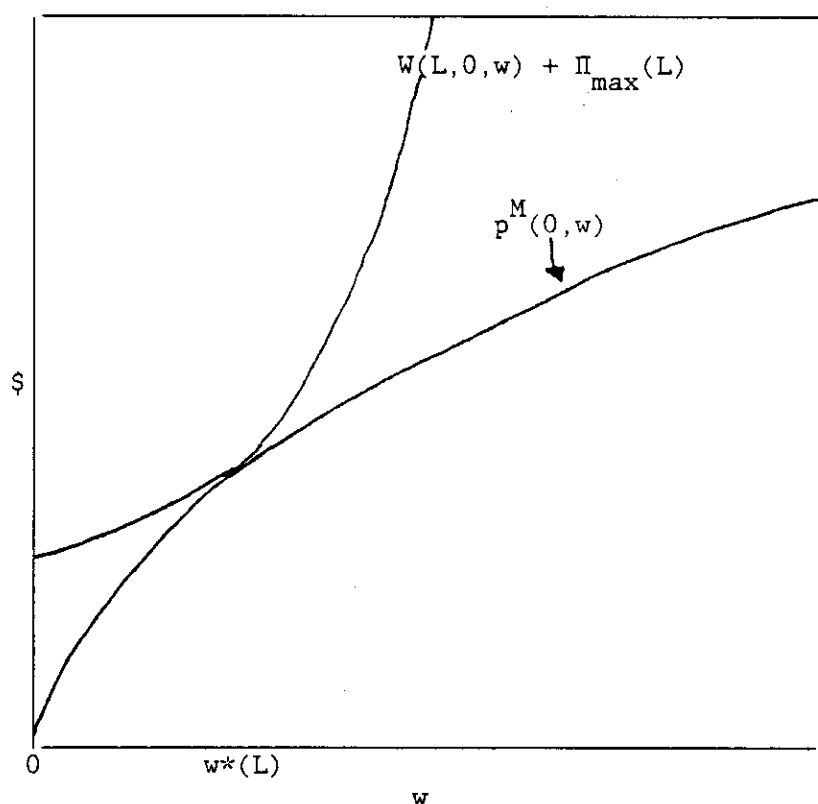
$$\begin{aligned} \text{where } r &= 0 \quad \text{if } q = L \\ r &= 1 \quad \text{if } q = H . \end{aligned}$$

We have established that $p^M(r, w)$, the consumer's reservation price, is strictly concave in the warranty. If it is also true that $W(q, r, w)$, the expected cost to the firm of the warranty, is a convex function of the warranty, the monopolist's profits are a strictly concave function of the

degree of warranty coverage.⁵ I will write the unique warranty that solves this known quality profit maximization problem as $w^*(q)$, and the maximum profit the firm earns when its quality is known as $\Pi_{\max}(q)$. In Figure 1, I graphically depict the problem faced if the product is low quality.⁶ The consumer's reservation prices are given by $p^M(0,w)$. The second curve shown is an iso-profit locus since consumer observes the product's low quality. The profit-maximizing warranty is $w^*(L)$.

Figure 1

Optimal Price and Warranty when Low Quality is Observable



⁵ A sufficient, although not necessary, condition for $\partial W^2(q,r,w)/\partial w^2 \geq 0$ is $\pi_{mmm}(q,m) \leq 0$ and $g'''(m) \leq 0$.

⁶ Figure 1 shows $w^*(q)$ as lying between 0 and θ . Corner solutions are possible, depending on the degree of moral hazard.

It may seem intuitive that if the monopolist produces a high quality product it will offer a higher warranty than if it produced a low quality product, that is, that $w^*(H) > w^*(L)$. In particular, we might expect this to be true when quality and effort are complements in determining the probability of success, since in this case warranty costs at any given w are less if the product is of high rather than low quality. However, a counter example shows that $w^*(L)$ may be greater than $w^*(H)$.

Suppose that $\pi(q,m) = q \cdot m$ and that $L = 0$ --this means that the low quality product will always break down. Using (4), $w^*(L)$ is defined by $h'(U(y) - U(w^*(L)))U'(w^*(L)) - 1 = 0$. Clearly, if $\partial p^M(1,w)/\partial w$ is less than $\partial W(H,1,w)/\partial w$ at $w = w^*(L)$, it must be true that $w^*(H) < w^*(L)$. Since $[\partial p^M(1,w)/\partial w] - [\partial W(H,1,w)/\partial w]$ can be rewritten as

$$(1-H \cdot m(1,w))(h'(U(y) - U(w) - H \cdot m(1,w)\Delta(w) + g(m(1,w)))U'(w)-1) \\ - \frac{H^2 U'(w)}{g''(m(1,w))} w ,$$

which is negative at $w = w^*(L)$, we can conclude that $w^*(H)$ is less than $w^*(L)$.

As long as quality is not directly observable by the consumer, the firm will have to take into account how its choice of price and warranty affects the consumer's beliefs. Since the consumer's reservation price $p^M(r,w)$ is increasing in r for all partial warranties, for a given warranty the firm can charge the consumer the highest price by convincing him that its product is high quality. If effort $m(r,w)$ is also increasing in r , then we can conclude that profits are monotonically increasing in beliefs. But if effort is decreasing in beliefs, then the consumer's reservation price and the expected warranty costs both increase as the

belief that the product is of high quality strengthens. This means that profits--at least when quality and effort are substitutes--need not be monotonic in the consumer's beliefs about the quality of the product.

It seems intuitively reasonable that quality and effort work as complements in determining the probability that some products will work successfully. Such products would be more sensitive to the level of effort taken if they are of high rather than low quality. Low quality units would be very likely to break down no matter what effort was taken; using them with great care would make breakdown only slightly less likely. However, there is no reason to believe that quality and effort are complements for all products. High quality units may easily be less sensitive to effort levels: they may withstand abuse more readily than low quality products. If this is true, then quality and effort are substitutes in determining the probability of breakdown; profits from the sale of such a product of fixed but unobservable quality may well be non-monotonic in the consumer's belief about product quality.

Whether quality and effort are substitutes or complements, multiple equilibria are to be expected. Almost any contract might be supportable as an equilibrium strategy. To draw more specific conclusions about how warranties are provided under asymmetric information, we would like to limit the set of equilibria. At the same time, we would like to know which --if any--of the sequential equilibria are "reasonable." Equilibria in games of this sort strongly depend on out-of-equilibrium beliefs, and certain kinds of beliefs may seem unreasonable. One way to refine the definition of sequential equilibrium is to establish a definition of "reasonable": restricting attention to only those equilibria supported by

reasonable beliefs can strongly limit the set of equilibria of the game.

A number of refinements of sequential equilibrium have been developed in the literature. In this paper I will use a refinement based on equilibrium concepts defined by Farrell (1986) and Grossman and Perry (1986). It was employed in a two-type signaling game by Gertner, Gibbons, and Scharfstein (1987). In this model the refinement can be defined as follows:

Definition: The Farrell-Grossman-Perry Refinement. An equilibrium in which the monopolist earns profits of Π^L if it produces a low quality product and Π^H if it produces a high quality product satisfies the Farrell-Grossman-Perry refinement if and only if there does not exist a contract (\bar{p}, \bar{w}) such that

$$(a) \quad \bar{p} \leq p^M(1, \bar{w}) \quad \text{while} \quad \Pi^H < \bar{p} - W(H, 1, \bar{w}) \quad \text{and} \quad \Pi^L > \bar{p} - W(L, 1, \bar{w})$$

or such that

$$(b) \quad \bar{p} \leq p^M(0, \bar{w}) \quad \text{while} \quad \Pi^H > \bar{p} - W(H, 0, \bar{w}) \quad \text{and} \quad \Pi^L < \bar{p} - W(L, 0, \bar{w})$$

or such that

$$(c) \quad \bar{p} \leq p^M(r^N, \bar{w}) \quad \text{while} \quad \Pi^H < \bar{p} - W(H, r^N, \bar{w}) \quad \text{and} \quad \Pi^L < \bar{p} - W(L, r^N, \bar{w}) .$$

The Farrell-Grossman-Perry refinement removes all equilibria that are vulnerable to what are termed consistent deviations--an equilibrium is "unreasonable" if a consistent deviation away from it exists. In this two-type model there are three kinds of consistent deviations. A consistent high-quality separating deviation is a contract (p, w) that yields the monopolist higher than equilibrium profits if and only if the monopolist produces high quality, as long as the consumer would conclude upon observing (p, w) that the monopolist must offer high quality. Part (a) of the

definition defines this kind of consistent deviation. The second type of deviation defined by part (b), is a consistent low quality separating deviation. Any contract that is such a deviation would yield the monopolist producing low quality higher than equilibrium profits--and if the monopolist produces high quality, lower than equilibrium profits--if the consumer would be convinced after observing this contract that the product is of low quality. The third type of deviation is a consistent pooling deviation, which is defined in part (c). Such a deviation yields higher than equilibrium profits, regardless of quality, if the consumer maintains his original belief that the product is high quality with probability r^N .

In the next section, I will analyze the signaling game under the assumption that $\pi_{qm} > 0$, so that quality and effort are complements and the firm's profits are increasing in consumer beliefs. I will establish that there exists no pooling equilibrium, and at most one separating equilibrium satisfying the Farrell-Grossman-Perry refinement. I will then give conditions under which the unique separating equilibrium exists. Then, in Section 4, I will analyze the game under the assumption that quality and effort are substitutes.

Section 3: Warranties When Consumer Effort and Unobservable Quality are Complements

When quality and effort are complements in determining the probability of breakdown, profits to the firm are monotonically increasing in beliefs, and the game played by the firm and the consumer is a variation on the signaling games familiar in the literature. The variation is that the consumer conditions beliefs on two observables--price and warranty--rather than

on a single variable, which makes this a multivariate signaling model similar to the one studied by Milgrom and Roberts (1986).

There are both multiple pooling and multiple separating equilibria to this game. This is the usual result in signaling models, but the plethora of equilibria makes it impossible to draw any conclusions about the effect of asymmetric information on warranty provision. Applying the Farrell-Grossman-Perry refinement removes this multiplicity. To begin with, no pooling equilibrium satisfies the refinement, as the following proposition demonstrates.

Proposition 1: Let (p, w) be a contract supportable as a pooling equilibrium to this game. Then there exists a contract (\bar{p}, \bar{w}) that is a consistent high quality separating deviation away from the equilibrium.

Proof: I need to show that there exists a (\bar{p}, \bar{w}) such that

- (i) $\bar{p} \leq p^M(1, \bar{w})$
- (ii) $p - W(H, r^N, w) < \bar{p} - W(H, 1, \bar{w})$, and
- (iii) $p - W(L, r^N, w) > \bar{p} - W(L, 1, \bar{w})$.

Re-arranging and combining (ii) and (iii) yields the following condition on (\bar{p}, \bar{w}) :

$$(iv) \quad W(L, r^N, w) - W(L, 1, \bar{w}) < p - \bar{p} < W(H, r^N, w) - W(H, 1, \bar{w}) .$$

Set $\bar{w} = w$, the pooling equilibrium warranty. Then if we can find a price $\bar{p} \leq p^M(1, w)$ such that (iv) is satisfied, we are done. Since $\pi_{qm} > 0$, $\partial m(r, w)/\partial r > 0$ as well, and these two facts taken together determine that

$$0 < W(L, r^N, w) - W(L, 1, w) < W(H, r^N, w) - W(H, 1, w) .$$

Let
$$\bar{p} = p - \frac{1}{2} \left\{ W(L, r^N, w) - W(L, 1, w) + W(H, r^N, w) - W(H, 1, w) \right\}.$$

Then $\bar{p} < p$ and $p < p^M(r^N, w)$, so that $\bar{p} < p^M(H, w)$. The other condition (iv) on (\bar{p}, w) is met by construction. Hence (\bar{p}, w) is a consistent high quality separating deviation. Q.E.D.

Any given pooling equilibrium can fail to satisfy the refinement for several reasons, since there are three kinds of consistent deviations. The proposition establishes that no pooling equilibrium can satisfy the refinement because a consistent high-quality separating deviation always exists. This deviation is a contract combining the pooling warranty with a lower price: the firm will deviate to this contract if and only if its product is of high quality. To establish the existence of this deviation, I used the assumption of complementarity: any given increase in effort will decrease total warranty costs, and the decrease is greater if quality is high than if it is low. Effort increases if the firm convinces the consumer that it sells a high quality product without changing the warranty. The firm would be willing to cut price from the pooling price to convince consumers of this, but it is willing to cut price by less if its product is low quality than it would be if its product were high quality. Hence there is a price cut that the firm will take to convince consumers if and only if it produces a high quality good.

Since the refinement removes all pooling equilibria, I can restrict the search for equilibria satisfying the refinement to separating equilibria. There are multiple separating equilibria to the game, but in all of them the monopolist offers the low quality product with warranty $w^*(L)$ at price $p^M(0, w^*(L))$, exactly the contract that would be offered with this

product if its quality were observable. The easiest way to see why this contract must be offered is to consider the profit earned by the monopolist who offers low quality in a separating equilibrium. Such a monopolist cannot earn more than $\Pi_{\max}(L)$, since the consumer correctly infers the quality of its product by observing the equilibrium contract. But offering its equilibrium contract cannot yield profits of less than $\Pi_{\max}(L)$, either: if it did yield less the monopolist would deviate to $(p^M(0, w^*(L)), w^*(L))$, since it can always earn at least $\Pi_{\max}(L)$ with this contract. The separating equilibrium payoff to the firm if the product is low quality must then be equal to $\Pi_{\max}(L)$, and $(p^M(0, w^*(L)), w^*(L))$ is the unique contract yielding this profit when the product is known to be low quality.

Multiple separating equilibria are still possible because there is a potential plethora of contracts that can be supported as separating equilibrium strategies to be offered if the monopolist's product is of high quality. Not all contracts can be so supported, of course. From the definition of sequential equilibrium we can give three conditions which must be met for a contract (p, w) to be supportable as a sequential separating equilibrium strategy for a high quality producer given that the low quality producer offers $(p^M(0, w^*(L)), w^*(L))$. First, the consumer must be willing to purchase the product at (p, w) if he is convinced that it is high quality: this implies that $p \leq p^M(1, w)$. Second, if the firm produces low quality, it must weakly prefer to offer the contract $(p^M(0, w^*(L)), w^*(L))$ and reveal its quality rather than offering (p, w) and being falsely taken to be a high quality producer: this implies that $p - W(L, 1, w) \leq \Pi_{\max}(L)$. Finally, the monopolist producing high quality must earn higher profits by

offering (p, w) and revealing its quality than it could earn at any other contract given consumer beliefs. A necessary condition for this to be true is that $p - W(H, 1, w) \geq \max_w p^M(0, w) - W(H, 0, w)$.⁷

Note that offering any given contract (p, w) meeting these three conditions could be supported as a separating equilibrium strategy for the monopolist producing high quality: specify that the consumer believes the product is of high quality upon observing this (p, w) and concludes that the product must be of low quality upon observing any other contract. However, we do not yet know which--if any--of these contracts can be supported in equilibria satisfying the Farrell-Grossman-Perry refinement. Our next step must be to examine the set of separating equilibria to see when consistent deviations are possible.

We need not be concerned with consistent separating deviations for the low quality firm--they cannot exist for any of these equilibria, since the monopolist will earn $\Pi_{\max}(L)$ in equilibrium if it produces low quality. That leaves us with consistent separating deviations for the high quality firm, and consistent pooling deviations. I will ignore the latter for the moment, and check this set of separating equilibria for the first kind of deviation.

Fix a contract offered by the monopolist who produces high quality in a separating equilibrium. If there exist no consistent separating deviations then the contract must solve the following problem:

⁷This last condition also guarantees that the monopolist will not deviate to $(p^M(0, w^*(L)), w^*(L))$ the separating equilibrium contract offered with a low quality product.

$$\max_{p,w} p - W(H,1,w) \quad (M1)$$

$$\text{subject to (1) } p \leq p^M(1,w)$$

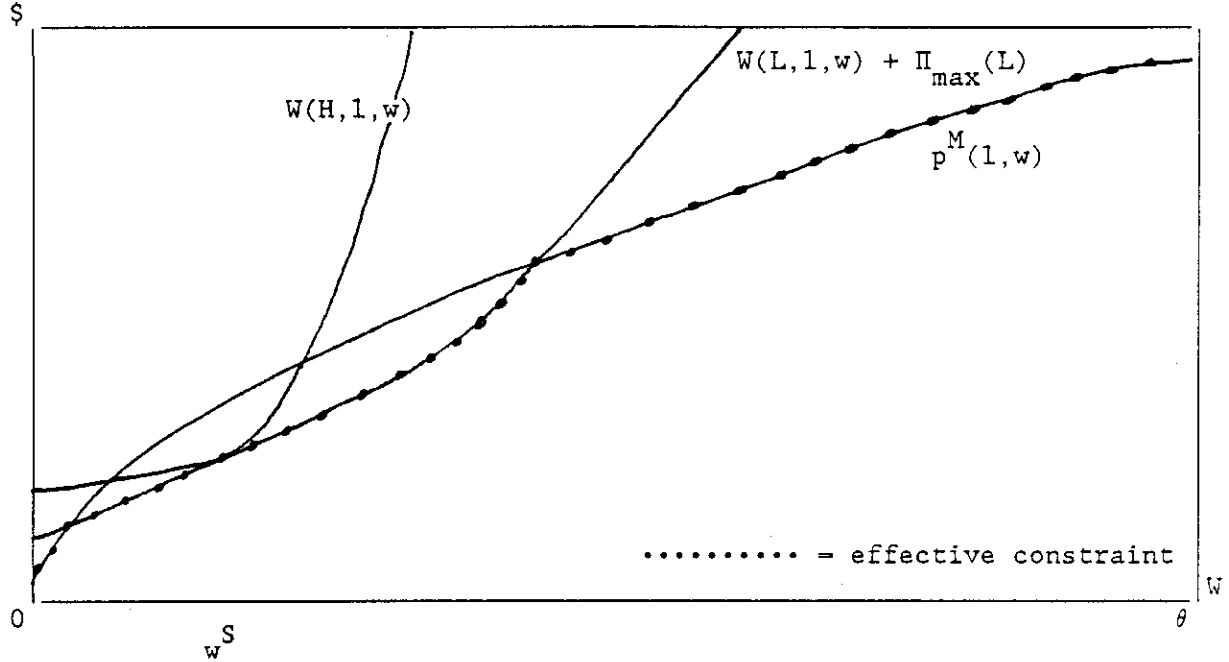
$$\text{and (2) } p - W(L,1,w) \leq \Pi_{\max}(L) .$$

If the contract did not solve this problem, then any contract that did would be a consistent high quality separating deviation. Solutions to (M1) maximize profits if the consumer is convinced that the product is high quality and attention is restricted to the set of contracts to which the monopolist would not deviate if it produced low quality. Note too that the two constraints guarantee that any contract solving this problem satisfies two of the three requirements already established as necessary for separating equilibrium. Hence, any contract satisfying this problem will be part of a sequential equilibrium solving the Farrell-Grossman-Perry refinement if a) no consistent pooling deviations exist, and

b) $p - W(H,1,w) \geq \max_w p^M(0,w) - W(H,0,w)$ so that the third of the necessary conditions is met.

Before we can check these last two requirements, we need to know about the contracts that solve our maximization problem (M1).

Figure 2: Contracts Solving the Problem



At any given warranty, w , profits are maximized by charging the highest price satisfying the constraints, the minimum of $(p^M(1,w), W(L,1,w) + \Pi_{\max}(L))$. Hence, the dotted line in Figure 2 is the effective constraint: maximizing subject to this constraint the profits to the monopolist who is correctly believed to produce high quality will give us the contracts we seek. As in Section 2, the iso-profit curves are of the form $p = W(H,1,w)$ plus a constant: we are looking for either a tangency between an iso-profit curve and the effective constraint or for a corner solution ($w = 0$ or $w = \theta$).

There are, in fact, four price-warranty contracts that are possible solutions to this problem, depending on whether one or the other or both of the constraints bind at the optimum. Table 1 displays the four possibilities:

Table 1
Contracts Solving (M1)

Contract	Solves (M1) Under These Conditions
$p^M(1, w^*(H)), w^*(H)$	$p^M(1, w^*(H)) - W(L, 1, w^*(H)) \leq \Pi_{\max}(L)$
$\Pi_{\max}(H) + W(L, 1, w^S), w^S$	$p^M(1, w^*(H)) - W(L, 1, w^*(H)) > \Pi_{\max}(L)$, and $p^M(1, w^S) \geq \Pi_{\max}(L) + W(L, 1, w^S)$
$p^M(1, w^3), w^3$	$p^M(1, w^*(H)) - W(L, 1, w^*(H)) > \Pi_{\max}(L)$, $p^M(1, w^S) < \Pi_{\max}(L) + W(L, 1, w^S)$, and $w^S < w^*(H)$
$p^M(1, w^4), w^4$	$p^M(1, w^*(H)) - W(L, 1, w^*(H)) > \Pi_{\max}(L)$, $p^M(1, w^S) < \Pi_{\max}(L) + W(L, 1, w^S)$, and $w^S > w^*(H)$

If only the first constraint is binding, the solution is

$(p^M(1, w^*(H)), w^*(H))$, the same contract that would be optimal if the consumer could directly observe the product's high quality.

If only the second constraint binds, the solution is

$(\Pi_{\max}(H) + W(L, 1, w^S), w^S)$ where w^S maximizes profits subject to the second constraint:

$$w^S = \operatorname{argmax}_w \Pi_{\max}(L) + W(L, 1, w) - W(H, 1, w)$$

Since $\partial^2 W(H, 1, w) / \partial w^2 > \partial^2 W(L, 1, w) / \partial w^2 > 0$, while $\partial W(H, 1, w) / \partial w$ is less than $\partial W(L, 1, w) / \partial w$ at $w = 0$ but greater than $\partial W(L, 1, w) / \partial w$ when $w = 1$, there is a unique w^S , determined by the tangency between $W(H, 1, w)$ and $W(L, 1, w)$. This solution is the one illustrated in Figure 2.

The two remaining solutions obtain when both constraints bind. The third is the left-hand intersection between the two constraints: the contract is $(p^M(1, w^3), w^3)$ where $w^3 = \min\{w | p^M(1, w) - \Pi_{\max}(L) + W(L, 1, w)\}$. This contract is preferred to the fourth possible solution when $w^S < w^*(H)$, since this is true if and only if $w^S < w^3$ and by definition w^S maximizes profits on the second constraint. The fourth and final contract that can solve (M1) is the right-hand intersection between the two constraints: the contract is $(p^M(1, w^4), w^4)$ where $w^4 = \max\{w | p^M(1, w) - \Pi_{\max}(L) + W(L, 1, w)\}$. This contract is preferred to the third possible solution when $w^S > w^*(H)$.

The solution to the maximization problem (M1), which I write as (p^*, w^*) , is clearly unique. Hence, we have established that there is at most one equilibrium satisfying the Farrell-Grossman-Perry refinement, an equilibrium in which the monopolist offers a high quality product with the contract (p^*, w^*) and the low quality product with the contract $(p^M(0, w^*(L)), w^*(L))$. However, we are not certain that this pair of contracts is supportable as a separating equilibrium, since in constructing (p^*, w^*) we ignored one of the three necessary conditions for this to be possible--namely, the high quality monopolist must earn more at (p^*, w^*) by revealing its quality than at any other contract given consumer beliefs. The contract (p^*, w^*) meets this ignored condition if and only if

$$p^* - W(H, 1, w^*) \geq \max_w p^M(0, w) - W(H, 0, w)$$

so that the monopolist will not deviate from its equilibrium strategy if it sells high quality and the consumer concludes that any contract other than (p^*, w^*) denotes low quality. If this condition is met we do indeed have a

separating equilibrium. We are still not certain that it satisfies the Farrell-Grossman-Perry refinement, although we do know that every other equilibrium violates the refinement. We have established that there are no consistent separating deviations away from this equilibrium, but we do not know if any consistent pooling deviations exist.

Consistent pooling deviations are defined by part (c) of the refinement, so that we need to know whether or not there exists a warranty \bar{w} such that

$$p^* - W(H, 1, w^*) < p^M(r^N, \bar{w}) - W(H, r^N, \bar{w}) \quad \text{and}$$

$$\Pi_{\max}(L) < p^M(r^N, \bar{w}) - W(L, r^N, \bar{w}) .$$

If such a warranty exists, then $(p^M(r^N, \bar{w}), \bar{w})$ is a consistent pooling deviation since the monopolist is willing to deviate to this contract regardless of quality if the consumer does not update his beliefs upon observing this contract.

There are warranties which satisfy the second of these inequalities. The obvious example is $w^*(L)$: the monopolist would be charging a higher price and paying lower warranty costs at $(p^M(r^N, w^*(L), w^*(L)))$ than at $(p^M(0, w^*(L), w^*(L)))$. Since there are contracts to which the monopolist would deviate to if it produces low quality and beliefs are not updated upon observing the deviation, the existence of a consistent pooling deviation depends on deviations by the monopolist produces high quality. There may not exist any warranty satisfying the first inequality: this is likely to be true for very low levels of r^N , situations in which the consumer is nearly certain that the product is of low quality before he observes the

contract offered with it. Even if there are warranties that satisfy the first inequality none may simultaneously satisfy the second. In this case, the monopolist is always willing to deviate to some pooling contract, but none of these potential deviations is consistent.

In summary, we now have established

Proposition 2: There exists a unique equilibrium satisfying the refinement if and only if two conditions are met:

$$(i) \quad p^* - W(H, 1, w^*) \geq \max_w p^M(0, w) - W(H, 0, w)$$

and \bar{w} such that

$$(ii) \quad p^* - W(H, 1, w^*) < p^M(r^N, \bar{w}) - W(H, r^N, \bar{w})$$

and $\Pi_{\max}(L) < p^M(r^N, \bar{w}) - W(L, r^N, \bar{w})$.

This is a separating equilibrium in which the contract (p^*, w^*) will be offered if the product is high quality and $(p^M(0, w^*(L)), w^*(L))$ will be offered if the product is low quality. If the necessary and sufficient conditions are not met, no equilibrium satisfies the refinement.

What then is the effect of asymmetric information on warranty provision in this game with consumer maintenance effort and quality as complements? Comparing the optimal contracts under observable quality with the separating equilibrium contracts yields some answers. To begin, it is possible that asymmetric information has no impact on warranty provision. If (p^*, w^*) equals $(p^M(1, w^*(H)), w^*(H))$, then when quality is unobserved the monopolist offers its product, whether of high or low quality, with the same contract as it would employ if quality were observable. And, in any case, asymmetries in information about quality can affect only the contract

offered if the product is high quality: a low quality product is always offered with the contract $(p^M(0, w^*(L)), w^*(L))$.

This model does not support the view that only higher warranties signal higher quality. It is the entire price-warranty contract that serves as a signal to consumers of product quality. The high-quality signaling warranty w^* may be greater or less than $w^*(H)$, the optimal observed high quality warranty, and in fact it can also be more or less extensive than $w^*(L)$, which is the equilibrium warranty offered with a low quality product. We do know that p^* , the price in the signaling contract, cannot be greater than $p^M(1, w^*(H))$ if w^* is less than $w^*(H)$.

An example illustrates that $w^* < w^*(H) < w^*(L)$ is a possible outcome. Let $\pi(m, q)$ be equal to $m \cdot q$ and let $L = 0$. We determined in Section 2 that under these conditions $w^*(H) < w^*(L)$. Assume also that $U(x) = \sqrt{x}$, $g(m) = m^2$, $y = 36$, $\theta = 4$, and $H = 1$. Then $\frac{\partial W(H, 1, w)}{\partial w} = \frac{3\sqrt{w}}{4}$, $\frac{\partial W(L, 1, w)}{\partial w} = 1$, and

$$\frac{\partial p^M(1, w)}{\partial w} = \left[6 - \sqrt{w} - \frac{(2 - \sqrt{w})^2}{4} \right] \left(\frac{1}{\sqrt{w}} - \frac{(2 - \sqrt{w})}{2\sqrt{w}} \right).$$

Solving for w^S shows that $w^S = 16/9$; $\partial p^M(1, w)/\partial w$ evaluated at w^S is greater than 1, implying that $w^S < w^*(H)$.

We have a clearer understanding of how information affects the payoffs to the firm and its consumers. Consumers always know the quality of any product they purchase: if that quality cannot be observed, they correctly infer it from the price-warranty contract. However, if they are offered a high quality product, they may be better off when quality is unobservable than they would be if offered an observably high quality product or a low quality (observable or not) product. When the unobservably high quality

product is offered with $(\Pi_{\max}(L) + W(L, 1, w^S), w^S)$ in equilibrium, the consumer improves upon his reservation utility by purchasing the product. The intuition is clear: the monopolist signals its high quality by forgoing profits, offering a non-optimal warranty at a price lower than the consumer's reservation price. It is the pricing that makes the consumer better off than he would be if he could observe quality. Of course, just because the high quality monopolist is forgoing profits to signal quality does not mean that signaling makes the consumer better off: $(p^M(1, w^3), w^3)$ and $(p^M(1, w^4), w^4)$ are both possible signaling equilibrium contracts and the consumer earns only his reservation utility from either.

The upshot is that the consumer always does at least as well when quality is unobservable as he could if it were observed before purchase. We also know if the firm produces low quality, its profits are the same whether or not quality is observable; if it produces high quality, then it never earns more under unobservable quality than it could if quality were known to consumers before purchase.

All of these results are contingent on two assumptions: first that quality and effort are in fact complements and second that an equilibrium satisfying the Farrell-Grossman-Perry refinement exists. In the next section, I will address the first assumption by analyzing the game when quality and effort are substitutes. As we will see, this game has some very different equilibria. I have established necessary and sufficient conditions for the second assumption to be met. It should also be noted that an alternative approach to refining sequential equilibrium, the combination of removing strategies from the game through iterated weak domination and applying the Cho-Kreps (1987) Intuitive Criterion yields exactly the same

unique equilibria under weaker conditions for existence: condition (i) of Proposition 3 must still hold but (ii) can be dropped.⁸

Section 4: Warranties When Consumer Effort and Unobserved Quality Are Substitutes

The analysis presented in the last section will not apply if higher product quality does not increase the marginal utility of an increase in effort. However, there is no compelling reason to think that quality and effort are generally complements. It seems entirely plausible that for many consumer products quality and effort are substitutes in determining the probability that the product will work. This would mean that higher quality decreases the marginal utility to the consumer of an increase in effort; the result is that $m(r, w)$ is decreasing in r . The consumer's reservation price $p^M(r, w)$ is increasing in r no matter what the relationship may be between quality and effort, but when they are substitutes, $W(q, r, w)$ is increasing in r as well. The implication is that the firm's profit at any given (p, w) with $p^M(0, w) < p \leq p^M(1, w)$ may be a non-monotonic function of the consumer's beliefs about product quality. It is no longer necessarily good for the product to be thought good; neither is it necessarily good for the product to be thought bad. This section analyzes the resulting game between firm and consumer.

If consumers cannot observe quality, the firm's incentives are more

⁸See Lutz (1986) for the application of this method to this game. The key to the equivalence between refinements in this game is the monotonicity of profits in r , consumer beliefs. Profits may be non-monotonic under the assumption in the next section that $\pi_{qm} < 0$: in this case equilibria satisfying the Intuitive Criterion may still be subject to consistent separating deviations.

complicated than in the previous section. There is still a possible incentive to convince consumers that the product is high quality, because consumers are willing to pay more if so convinced. But there is now also a possible incentive to convince consumers that the product is low quality, since doing so would minimize warranty costs by leading the consumer to take a large amount of effort.

These mixed incentives can be expected to complicate equilibrium behavior. A pooling equilibrium might be supported by off-the-equilibrium path beliefs that make it too costly or impossible for the monopolist to reveal low quality. There may be separating equilibria in which the monopolist earns observable quality profits if it produces high quality but earns less than observable quality profits if its product is low quality to signal that quality. There can also be separating equilibria in which the monopolist always earns less profit than it could if its quality were observable.

All these kinds of equilibria can exist in the same game. But limiting our attention to only those equilibria that satisfy the Farrell-Grossman-Perry refinement removes most of these equilibria from consideration. We begin by applying the refinement to the game's separating equilibria.

By definition, a separating equilibrium in which the firm offers a warranty w^q if it produces quality q cannot satisfy the refinement if there are consistent separating deviations away from the equilibrium. This means that a separating equilibrium with contracts (\hat{p}^H, \hat{w}^H) and (\hat{p}^L, \hat{w}^L) can only satisfy the refinement if

$$(\hat{p}^H, \hat{w}^H) = \operatorname{argmax}_{p, w} p - W(H, 1, w) \quad (M2)$$

subject to (1) $p \leq p^M(1, w)$

$$(2) \quad p - W(L, 1, w) \leq \hat{p}^L - W(L, 0, \hat{w}^L)$$

since otherwise a consistent high-quality separating deviation will exist.

At the same time the separating equilibrium contracts must solve

$$(\hat{p}^L, \hat{w}^L) = \operatorname{argmax}_{p, w} p - W(L, 0, w) \quad (M3)$$

subject to (1) $p \leq p^M(0, w)$

$$(2) \quad p - W(H, 0, w) \leq \hat{p}^H - W(H, 1, \hat{w}^H)$$

since otherwise a consistent low quality separating deviation will exist.

A separating equilibrium to this game satisfies the refinement if and only if (a) the equilibrium contracts simultaneously solve (M2) and (M3) and (b) there are no consistent pooling deviations. There is at most a single separating equilibrium satisfying the refinement. We leave the proof of the next proposition to the Appendix.

Proposition 3: There is a unique pair of contracts (\hat{p}^H, \hat{w}^H) and (\hat{p}^L, \hat{w}^L) that simultaneously solves (M2) and (M3). These contracts are supportable in a separating equilibrium satisfying the refinement if and only if

$$(1) \quad \hat{\Pi}^H > \max_w p^M(0, w) - W(H, 1, w),$$

$$(2) \quad \hat{\Pi}^L > \max_w p^M(0, w) - W(L, 1, w), \text{ and}$$

(3) There is no contract $(p^M(r^N, w^P), w^P)$ such that

$$(a) \quad w^P \in \Omega = \{w : \partial W(L, r^N, w)/\partial w \geq \partial p^M(r^N, w)/\partial w \geq \partial W(H, r^N, w)/\partial w\}$$

$$(b) \quad p^M(r^N, w^P) - W(L, r^N, w^P) > \hat{\Pi}^L$$

$$(c) \quad p^M(r^N, w^P) - W(H, r^N, w^P) > \hat{\Pi}^H.$$

Conditions (1) and (2) establish that a separating equilibrium exists in which (\hat{p}^q, \hat{w}^q) is the contract offered when quality is q . Condition (3) establishes that there are no consistent pooling deviations from the equilibrium.

This separating equilibrium differs from the one discussed in Section 3 in two interesting ways. First, it need not be the high quality monopolist who manipulates its contract to convince consumers of its quality, while the low quality good is sold at $(p^M(0, w^*(L)), w^*(L))$. If $\Pi_{\max}(H) < p^M(0, w^*(L)) - W(H, 0, w^*(L))$ and $\Pi_{\max}(L) \geq p^M(1, w^*(H)) - W(L, 1, w^*(H))$, this familiar pattern still holds. But if these two inequalities are reversed, it is the low quality producer who manipulates its equilibrium contract: the high quality good would be sold at $(p^M(1, w^*(H)), w^*(H))$. And the firm may signal regardless of quality: this will happen when $\Pi_{\max}(L) < p^M(1, w^*(H)) - W(L, 1, w^*(H))$ and $\Pi_{\max}(H) < p^M(0, w^*(L)) - W(H, 0, w^*(L))$.

Secondly, in this case high quality is signalled with a warranty \hat{w}^H that is more extensive than $w^*(H)$, the warranty the firm would offer if this quality were observable. Low quality is signalled with a warranty \hat{w}^L which is less than $w^*(L)$. In fact, if $\hat{\Pi}^L \leq p^M(1, \theta) - W(L, 1, \theta)$ and $\hat{\Pi}^H \leq p^M(0, 0) - W(H, 0, 0)$, $\hat{w}^H = \theta$ and $\hat{w}^L = 0$, so that the low quality product is offered with no warranty and the high quality product is sold with a full warranty. This relationship holds because $\partial W(L, r, w)/\partial w$ is greater than $\partial W(H, r, w)/\partial w$ for all w and r . If the second constraint in (M2) is binding at (\hat{p}^H, \hat{w}^H) and (\hat{p}^L, \hat{w}^L) , then \hat{w}^H must be greater

than $w^*(H)$. In the same way, if the second constraint in (M3) is binding at these contracts, then \hat{w}^L must be less than $w^*(L)$. Recalling that $w^*(L)$ may be greater than $w^*(H)$, we can see that \hat{w}^L may be greater than \hat{w}^H .

In the previous section, we showed that under the assumption that quality and effort are complements, no pooling equilibrium satisfied the refinement; there was always a consistent separating deviation away from any such equilibrium. This result does not hold if quality and effort are substitutes. The next proposition gives both necessary and sufficient conditions for a pooling equilibrium to satisfy the refinement; the proof is left to the Appendix.

Proposition 4: Necessary conditions for a pooling equilibrium on a contract (p^P, w^P) which yields payoff $\bar{\pi}^q = p^P - W(q, r^N, w^P)$ if the monopolist produces quality q , to satisfy the refinement are a) $p^P = p^M(r^N, w^P)$, b) $w^P \in \Omega = \{w : \partial W(L, r^N, w)/\partial w \geq \partial p^M(r^N, w)/\partial w \geq \partial W(H, r^N, w)/\partial w\}$ and c) $\bar{\pi}^q \geq \hat{\pi}^q$ for all q . Sufficient conditions are a), b), and d) $\bar{\pi}^q \geq \Pi_{\max}(q)$ for all q .

Although only a single separating equilibrium can satisfy the refinement, multiple pooling equilibria may do so. The existence of one kind of equilibrium does not rule out the existence of the other. There might be a separating equilibrium and a pooling equilibrium, both satisfying the refinement, such that $\bar{\pi}^L = \hat{\pi}^L$ and $\bar{\pi}^H > \hat{\pi}^H$, or a pair of equilibria with $\bar{\pi}^L > \hat{\pi}^L$ and $\bar{\pi}^H = \hat{\pi}^L$, or a pair that are payoff-equivalent. If more than two equilibria satisfy the refinement, they are

all pooling equilibria.

In pooling equilibria consumers pay more for a low quality product than they would be willing to pay if they could observe quality: they pay $p^M(r^N, w^P)$, which is greater than $p^M(0, w^P)$. But they are actually no worse off than they would be if quality were observable, since they take effort $m(r^N, w^P)$ which is less than $m(0, w^P)$. In the same way, if the product is high quality, consumers pay less when they cannot infer quality than they would be willing to pay if that quality were observable. But they also take more effort and are, overall, no better off.

The firm, on the other hand, whether it produces high or low quality might be able to earn higher profits from pooling than it could if its quality were observable. Not surprisingly, if this is true, then pooling will result --but this is not a necessary condition for the existence of a pooling equilibrium.

Section 5: Conclusion

The model I have analyzed is quite specialized. Some of the assumptions can be generalized: these include the assumptions of identical consumers and the lack of any income effect on effort. Others are key to the analysis, chiefly the assumption that the product is sold by a single firm, which limits the applicability of this model to many of the markets in which warranties are observed. Most consumer appliance markets, for example, are oligopolistic. However, we have found no support for the idea that signaling necessarily implies that higher warranties mean higher quality. And it is clear that if warranties do serve as signals, they do so together with price--and perhaps also in combination with other

available signals, "money-burning" and the like.

The incentives affecting the consumer's choice of effort can have an important effect on the predictions of the model. Whether or not pooling equilibria exist, and whether or not the monopolist's price-warranty contract might be expected to change if quality became observable, depends in part on derivatives of the expected breakdown function. Hence, the role of warranties will have to be addressed on a market by market basis, because these derivatives reflect technological factors about the product, and quality and effort can be expected to be complements in some markets and substitutes in others.

Information about how quality and effort determine the likelihood that the product works may also be important in addressing policy questions. If quality and effort are complements in equilibrium consumers infer product quality even when they can not observe it, and because they value both signals (high warranty/low price) they are at least as well off when they cannot observe quality as they would be if they could do so before purchase. There is little reason to expect that consumers are somehow exploited through warranties and good reason to expect that any manufacturer of a product of unobservably high quality is interested in ways of making that quality observable. When quality and effort are substitutes, consumers are again at least as well off under any equilibrium as they would be if they had complete information about product quality. But they may not be able to infer product quality from the equilibrium price and warranty contract, and the monopolist may have no incentive to make any level of product quality observable.

APPENDIX

Proof of Proposition 3

We begin by defining

$$B_H(\Pi^L) = \max_{p,w} p - W(H,1,w)$$

$$\text{subject to } p \leq \min\{p^M(1,w), \Pi^L + W(L,1,w)\}$$

and

$$B_L(\Pi^H) = \max_{p,w} p - W(L,0,w)$$

$$\text{subject to } p \leq \min\{p^M(0,w), \Pi^H + W(H,0,w)\} .$$

If there exists a $\hat{\Pi}^H$ such that $\hat{\Pi}^H = B_H(B_L(\hat{\Pi}^H))$ then there is no consistent separating deviation from an equilibrium in which the monopolist earns $\hat{\Pi}^q$ by selling the quality q product, with $\hat{\Pi}^L = B_L(\hat{\Pi}^H)$. We show that such $\hat{\Pi}^H$ and $\hat{\Pi}^L$ exist and are unique.

Solving for $B_H(\Pi^L)$ yields

$$B_H(\Pi^L) = \Pi^L + W(L,1,\theta) - W(H,1,\theta)$$

$$\text{if } 0 \leq \Pi^L < p^M(1,\theta) - W(L,1,\theta)$$

$$= \Pi^L + W(L,1,\bar{w}(\Pi^L)) - W(H,1,\bar{w}(\Pi^L))$$

$$\text{if } p^M(1,\theta) - W(L,1,\theta) \leq \Pi^L \leq p^M(1,w^*(H)) - W(L,1,w^*(H))$$

$$= \Pi_{\max}^H(H)$$

$$\text{if } \Pi^L > p^M(1,w^*(H)) - W(L,1,w^*(H))$$

$$\text{where } \bar{w}(\Pi^L) = \max\{w : p^M(1,w) - W(L,1,w) = \Pi^L\} .$$

Note that $B_H(\Pi^L)$ is a continuous and weakly increasing function of Π^L , mapping $[0, \Pi_{\max}^L(L)]$ into $[0, \Pi_{\max}^L(H)]$. It is piecewise differentiable and $0 \leq dB_H(\Pi^L)/d\Pi^L \leq 1$.

Solving for $B_L(\Pi^H)$ yields

$$\begin{aligned} B_L(\Pi^H) &= \Pi^H \quad \text{if } 0 \leq \Pi^H < p^M(0,0) \\ &= \Pi^H + W(H,0,\underline{w}(\Pi^H)) \\ &\quad \text{if } p^M(0,0) \leq \Pi^H \leq p^M(0,w^*(L)) - W(H,0,w^*(L)) \\ &= \Pi_{\max}^L(L) \\ &\quad \text{if } \Pi^H > p^M(0,w^*(L)) - W(H,0,w^*(L)) \end{aligned}$$

$$\text{where } \underline{w}(\Pi^H) = \min\{w : p^M(0,w) - W(H,0,w) = \Pi^H\}.$$

Note that $B_L(\Pi^H)$ is a continuous and weakly increasing function of Π^H , mapping $[0, \Pi_{\max}^H(H)]$ into $[0, \Pi_{\max}^H(L)]$. It is piecewise differentiable and $0 \leq dB_L(\Pi^H)/d\Pi^H \leq 1$.

By Brouwer's fixed point theorem, the composite function $B_H(B_L(\Pi^H))$ has a fixed point, since it is continuous and maps $[0, \Pi_{\max}^H(H)]$ into itself. Let $\hat{\Pi}^H$ be a fixed point of the composite function. The fixed point is unique: since $dB_H(\Pi^L)/d\Pi^L \leq 1/[dB_L(\Pi^H)/d\Pi^H]$, $B_H(B_L(\Pi^H))$ is less than Π^H for any $\Pi^H > \hat{\Pi}^H$ and greater than Π^H for any $\Pi^H < \hat{\Pi}^H$.

There is a unique (\hat{p}^H, \hat{w}^H) such that $\hat{p}^H - W(H,1,\hat{w}^H) = \hat{\Pi}^H$, $\hat{p}^H \leq p^M(1,\hat{w}^H)$ and $\hat{p}^H - W(L,1,\hat{w}^H) \leq \hat{\Pi}^L$. We prove this by noting that if there is a second contract $(\tilde{p}^H, \tilde{w}^H)$ such that $\tilde{p}^H - W(H,1,\tilde{w}^H) = \hat{\Pi}^H$, $\tilde{p}^H \leq p^M(1,\tilde{w}^H)$, and $\tilde{p}^H - W(L,1,\tilde{w}^H) \leq \hat{\Pi}^L$, any convex combination of the two contracts, say (p,w) , would satisfy $p \leq p^M(1,w)$, and $p - W(L,1,w) \leq \hat{\Pi}^L$, while $p - W(H,1,w)$ would be greater than $\hat{\Pi}^H$. This

is true because $p^M(1,w)$ is increasing and strictly concave, while $W(q,1,w)$ is increasing with $\partial W(H,1,w)/\partial w < \partial W(L,1,w)/\partial w$. But then (p,w) would show that $\hat{\Pi}^H \neq B_H(\hat{\Pi}^L)$, and this is a contradiction. The parallel argument shows that because $p^M(0,w)$ is increasing and concave and $0 < \partial W(H,0,w)/\partial w < \partial W(L,0,w)/\partial w$, there is a unique contract (\hat{p}^L, \hat{w}^L) such that $\hat{p}^L - W(L,0,\hat{w}^L) = \hat{\Pi}^L$, $\hat{p} \leq p^M(0,\hat{w}^L)$, and $\hat{p} - W(H,0,\hat{w}) \leq \hat{\Pi}^H$.

We have not established yet that a separating equilibrium satisfying the refinement can exist; we have so far only proven that if it exists the monopolist's equilibrium strategy is unique. We need to see if (\hat{p}^H, \hat{w}^H) and (\hat{p}^L, \hat{w}^L) are supportable as separating equilibrium strategies; and if they are so supportable we need to see if there are any consistent pooling deviations away from the equilibrium.

By construction, $\hat{\Pi}^H \geq \hat{p}^L - W(H,0,\hat{w}^L)$ and $\hat{\Pi}^L \geq \hat{p}^H - W(L,1,\hat{w}^H)$, so we know that there is a separating equilibrium in which (\hat{p}^H, \hat{w}^H) is the contract offered if the product is high quality and (\hat{p}^L, \hat{w}^L) is the contract offered if the product is low quality as long as consumer beliefs give the monopolist no incentive to deviate to some out-of-equilibrium contract. Out of equilibrium payoffs are minimized, since $\partial p^M(r,w)/\partial r > 0$ and $\partial m(r,w)/\partial r < 0$, when beliefs are structured in the following way:

$$\begin{aligned} r(p,w) &= 0 \quad \text{for all } (p,w) \text{ such that } p > p^M(0,w) \\ &\quad \text{and } p \neq \hat{p}^H, \quad w \neq \hat{w}^H \\ r(p,w) &= 1 \quad \text{for all } (p,w) \text{ such that } p \leq p^M(0,w) \\ &\quad \text{and } p \neq \hat{p}^L, \quad w \neq \hat{w}^L. \end{aligned}$$

Thus (\hat{p}^H, \hat{w}^H) and (\hat{p}^L, \hat{w}^L) can be supported as a separating equilibrium if and only if

$$\hat{\Pi}^H > \max_w p^M(0, w) - W(H, 1, w)$$

$$\hat{\Pi}^L > \max_w p^M(0, w) - W(L, 1, w) .$$

Given that (\hat{p}^H, \hat{w}^H) and (\hat{p}^L, \hat{w}^L) are supportable as separating equilibrium strategies, what remains to be shown is that there is no pooling deviation from this equilibrium if and only if there is no contract $(p^M(r^N, w^P), w^P)$ where $w^P \in \Omega = \{w : \partial W(L, r^N, w)/\partial w \geq \partial p^M(r^N, w)/\partial w \geq \partial W(H, r^N, w)/\partial w\}$ such that $p^M(r^N, w^P) - W(L, r^N, w^P) > \hat{\Pi}^L$ while $p^M(r^N, w^P) - W(H, r^N, w^P) > \hat{\Pi}^H$.

The condition is clearly necessary; any such $(p^M(r^N, w^P), w^P)$ contract is a consistent pooling deviation.

The condition is sufficient because, for any (p, w) , there is a $w^P \in \Omega$ such that profits to the monopolist from offering (p, w) when the consumer holds beliefs r^N are regardless of quality no greater than the profits from offering $(p^M(r^N, w^P), w^P)$ when the consumer holds the same beliefs. Q.E.D.

Proof of Proposition 4

Conditions a) and b) are necessary and sufficient for there to be no consistent pooling deviations from the equilibrium. Condition d) guarantees that there is no consistent separating deviation from the equilibrium, since there is no contract $(p^M(0, w), w)$ such that $p^M(0, w) - W(L, 0, w) > \hat{\Pi}^L$ or contract $(p^M(1, w), w)$ such that $p^M(1, w) - W(H, 1, w) > \hat{\Pi}^H$.

What remains to be shown is that a consistent separating deviation will exist if $\hat{\Pi}^L < \hat{\Pi}^L$ or if $\hat{\Pi}^H < \hat{\Pi}^H$. If $\hat{\Pi}^L < \hat{\Pi}^L$ while $\hat{\Pi}^H \geq \hat{\Pi}^H$, then it follows that $B_L(\hat{\Pi}^H)$ must be greater than $\hat{\Pi}^L$, since B_L is

non-decreasing in $\tilde{\Pi}^H$ and $\hat{\Pi}^L = B_L(\hat{\Pi}^H)$. This means that there is a consistent low quality separating deviation from the pooling equilibrium. If $\tilde{\Pi}^H < \hat{\Pi}^H$ and $\tilde{\Pi}^L \geq \hat{\Pi}^L$, we conclude that $B_H(\tilde{\Pi}^L) > \tilde{\Pi}^H$. But then there is a consistent high quality separating deviation from the pooling equilibrium. Suppose finally that $\tilde{\Pi}^H \leq \hat{\Pi}^H$ and $\tilde{\Pi}^L \leq \hat{\Pi}^L$, with one of these inequalities being strict. Unless $B_L(\tilde{\Pi}^H) \leq \tilde{\Pi}^L$ while $B_H(\tilde{\Pi}^L) \leq \tilde{\Pi}^H$, a consistent separating deviation from the equilibrium will exist. If $\tilde{\Pi}^H < \hat{\Pi}^H$, then $B_H(B_L(\tilde{\Pi}^L)) > \tilde{\Pi}^H$. But this means that if $B_L(\tilde{\Pi}^H) \leq \tilde{\Pi}^L$, it must also be true that $B_H(\tilde{\Pi}^L) > \tilde{\Pi}^H$, since B_H is non-decreasing in Π^L . Similarly, if $\tilde{\Pi}^L < \hat{\Pi}^L$, then $B_L(B_H(\tilde{\Pi}^L)) > \tilde{\Pi}^L$ --but this means that $B_L(\tilde{\Pi}^H)$ must be greater than $\tilde{\Pi}^L$ while $B_H(\tilde{\Pi}^L) \leq \tilde{\Pi}^H$. Thus a consistent separating deviation from the pooling equilibrium must exist. Q.E.D.

References

- Bryant, W. Keith and Jennifer Gerner, "The Price of a Warranty: The Case for Refrigerators," Journal of Consumer Affairs (Spring 1978), 30-47.
- Cho, In-Koo, and David M. Kreps, "Signaling Games and Stable Equilibria," Quarterly Journal of Economics 52 (May 1987), 179-221.
- Cooper, Russell, and Thomas W. Ross, "Product Warranties and Double Moral Hazard," Rand Journal of Economics 16 (Spring 1985), 103-113.
- Farrell, Joseph, "Meaning and Credibility in Cheap-Talk Games," mimeo, GTE Labs, 1986.
- Gerner, Jennifer, and W. Keith Bryant, "Appliance Warranties as a Market Signal?" Journal of Consumer Affairs (Summer 1981), 75-86.
- Gertner, Robert, Robert Gibbons, and David Scharfstein, "Simultaneous Signaling to the Capital and Product Markets," mimeo, Massachusetts Institute of Technology, revised February 1987.
- Grossman, Sanford, "The Informational Role of Warranties and Private Disclosure about Product Quality," Journal of Law and Economics 24 (December 1981), 461-484.
- Grossman, Sanford, and Motty Perry, "Perfect Sequential Equilibrium," Journal of Economic Theory 39 (1986), 97-119.
- Heal, G. K., "Guarantees and Risk Sharing," Review of Economic Studies 44 (1977), 549-560.
- Lutz, Nancy A., "Warranties and Consumer Moral Hazard," unpublished Ph.D. Dissertation, Stanford University Graduate School of Business, November 1986.
- Matthews, Steven, and John Moore, "Monopoly Provision of Quality and Warranties: An Exploration in the Theory of Multi-Dimensional Screening," Econometrica 55 (March 1987), 441-468.
- Milgrom, Paul, and John Roberts, "Price and Advertising Signals of Product Quality," Journal of Political Economy 94 (August 1986), 796-824.
- Priest, George L., "A Theory of Consumer Product Warranty," Yale Law Journal 90 (May 1981), 1297-1352.
- Spence, A. Michael, "Job Market Signaling," Quarterly Journal of Economics 87 (1973), 355-374.